

Comparing the broken $U3 \times U3$ linear sigma model with experiment

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Abstract. The linear σ model with broken $U3 \times U3$ is compared with data of the lightest scalar and pseudoscalar mesons. When 5 of the 6 parameters are fixed by the pseudoscalar masses and decay constants, one finds that, already at the tree level, one has a reasonable description for the 4 scalar masses, describing mixing and up to 8 trilinear couplings of lightest scalars, taken as $a_0(980)$, $f_0(980)$, $\sigma(\approx 500)$ and $K_0^*(1430)$. This clearly indicates that these scalars are the chiral partners of the π , η , η' , K , and this strongly suggests that they, like the latter, are (unitarized) $q\bar{q}$ states.

1 Introduction

As is well known the naive quark models (NQM) fails badly in trying to understand the lightest scalars, the $a_0(980)$, $f_0(980)$, $K_0^*(1430)$ and the $\sigma(400-1200)$, which we shall here call $\sigma(500)$. Therefore, today most authors want to give the $a_0(980)$, $f_0(980)$ and the $\sigma(500)$ other interpretations than being $q\bar{q}$ states. Popular alternative interpretations are $K\bar{K}$ bound states, 4 quark states, or for the σ , a glueball. But there is also an obvious reason for why the NQM fails: Chiral symmetry is absent in the NQM, but is crucial for the scalars. Chiral symmetry is widely believed to be broken in the vacuum, and two of the scalars (σ and f_0) have the same quantum numbers as the vacuum. Thus to understand the scalar nonet in the same way as we believe we understand the vectors and heavier multiplets, and to make a sensible comparison with experiment, one must include chiral symmetry in addition to flavor symmetry into the quark model.

The simplest such chiral quark model is the linear $U3 \times U3$ sigma model with 3 flavors. In this case we can treat both the scalar and pseudoscalar nonets simultaneously and on the same footing, automatically getting small masses for the pseudoscalar octet, and symmetry breaking through the vacuum expectation values (VEV's) of the scalar fields.

As an extra bonus we have in principle a renormalizable theory, i.e. “unitarity corrections” are calculable. In fact, in the flavor symmetric ($u = d = s$ below) limit many of the unitarity corrections can be considered as being already included into the mass parameters of the theory, once the original 4–5 parameters are replaced by the 4 physical masses for the singlet and octet 0^{-+} and 0^{++} masses, and the pseudoscalar decay constant.

Unfortunately this model, over 30 years old, [1] has had very few recent phenomenological applications, although important exceptions are the intensive efforts of Scadron



Fig. 1. Graphical quark line representation of the λ and λ' terms of (1)

and collaborators [2]. The reestablishment [3, 4] of the light and broad σ has also more recently revitalized the interest in the linear sigma model [5–7].

2 The linear sigma model with 3 flavors

The well-known linear sigma model [1] generalized to 3 flavors with complete scalar (s_a) and pseudoscalar (p_a) nonets has at the tree level in the Lagrangian the same flavor and chiral symmetries as massless QCD. The $U3 \times U3$ Lagrangian with a symmetry breaking term \mathcal{L}_{SB} is

$$\mathcal{L} = \frac{1}{2} \text{Tr} [\partial_\mu \Sigma \partial_\mu \Sigma^\dagger] - \frac{1}{2} \mu^2 \text{Tr} [\Sigma \Sigma^\dagger] - \lambda \text{Tr} [\Sigma \Sigma^\dagger \Sigma \Sigma^\dagger] - \lambda' (\text{Tr} [\Sigma \Sigma^\dagger])^2 + \mathcal{L}_{\text{SB}}. \quad (1)$$

Here Σ is a 3×3 complex matrix, $\Sigma = S + iP = \sum_{a=0}^8 (s_a + ip_a) \lambda_a / 2^{1/2}$, in which λ_a are the Gell-Mann matrices, normalized as $\text{Tr} [\lambda_a \lambda_b] = 2 \delta_{ab}$, and where for the singlet $\lambda_0 = (2/N_f)^{1/2} \mathbf{1}$ is included. Each meson in (1) has a definite $SU3_f$ symmetry content, which in the quark model means that it has the same flavor structure as a $q\bar{q}$ meson. Thus the fields s_a and p_a and potential terms in (1) can be given a conventional quark line structure [8] (Fig. 1).

The symmetry breaking terms are most simply:

$$\mathcal{L}_{\text{SB}} = \epsilon_\sigma \sigma_{u\bar{u}+d\bar{d}} + \epsilon_{s\bar{s}} \sigma_{s\bar{s}} + \beta [\det \Sigma + \det \Sigma^\dagger], \quad (2)$$

which gives the pseudoscalars mass and break the flavor and $U_A(1)$ symmetries. The stability condition, that the linear terms in the fields must vanish after the shift of the scalar fields ($\Sigma \rightarrow \Sigma + V$), determines the small parameters ϵ_i in terms of the pion and kaon masses and decay constants. One finds $\epsilon_\sigma = m_\pi^2 f_\pi$, $\epsilon_{s\bar{s}} = (2m_K^2 f_K - m_\pi^2 f_\pi)/2^{1/2}$, while β in the $U_A(1)$ breaking term is determined by $m_{\eta'}$, or by $m_\eta^2 + m_{\eta'}^2$.

My fit to the scalars with the unitarized quark model (UQM) [9] is essentially a unitarization of (1) with $\lambda \approx 16$ and $\lambda' = 0$, and with the main symmetry breaking generated by putting the pseudoscalar masses at their physical values. The model was used as an effective theory with a symmetric smooth 3-momentum cutoff $0.54 \text{ GeV}/c$ given by a gaussian form factor. Such a form factor is natural, since physical mesons are of course not pointlike, but have a finite size of $0.7\text{--}0.8 \text{ fm}$. (See the discussion in connection to (37) below.) The fit included the Adler zeroes which follow from (1), but only approximates the crossing symmetry.

Here I shall study the theory at the tree level, leaving a detailed discussion of the unitarization for future work. In fact, when tadpole loops are included in the unitarization the “unitarity corrections” to the masses should not be too large, since the tadpole loops partly cancel the ($\log \Lambda$ divergence in) s -channel hadron loops. One expects the corrections to the mass spectrum to be at most of the same order as the flavor symmetry breaking. Because of the renormalizability, one can in the flavor symmetric limit include the unitarity corrections into the mass parameters, which are fixed by experiment.

Equation (1) without \mathcal{L}_{SB} is clearly invariant under $\Sigma \rightarrow U_L \Sigma U_R^\dagger$ of $U3 \times U3$. After shifting the flavorless scalar fields by the VEV’s ($\Sigma \rightarrow \Sigma + V$) to the minimum of the potential, the scalars acquire masses and also the pseudoscalars obtain a (small) mass because of \mathcal{L}_{SB} . Then the λ and λ' terms generate trilinear spp and sss couplings, in addition to those coming from the $U_A(1)$ symmetry breaking determinant term. The λ term, which turns out to be the largest, obeys the OZI rule, while the λ' and β terms violate this rule.

3 Tree-level masses

It is an ideal problem for a symbolic program like Maple V to calculate the predicted masses and the couplings from the Lagrangian. It has 6 parameters, $\mu, \lambda, \lambda', \beta, u = d$ and s , of which the last two define the diagonal matrix V with the flavorless meson VEV’s: $V = \text{diag}[u, d, s]$. These are at the tree level related to the pion and kaon decay constants through $u = d = \langle \sigma_{u\bar{u}, d\bar{d}} \rangle / 2^{1/2} = f_\pi / 2^{1/2}$ (assuming exact isospin symmetry) and $s = \langle \sigma_{s\bar{s}} \rangle = (2f_K - f_\pi) / 2^{1/2}$. One finds, denoting the often occurring combination $\mu^2 + 4\lambda'(u^2 + d^2 + s^2)$ by $\bar{\mu}^2$, and expressing the flavorless mass matrices in the ideally mixed frame,

$$m_{\pi^+}^2 = \bar{\mu}^2 + 4\lambda(u^2 + d^2 - ud) + 2\beta s, \quad (3)$$

$$m_{K^+}^2 = \bar{\mu}^2 + 4\lambda(u^2 + s^2 - su) + 2\beta d, \quad (4)$$

$$\begin{pmatrix} m_\eta^2 \\ m_{\eta'}^2 \end{pmatrix} = \text{eigv} \begin{pmatrix} \bar{\mu}^2 + 2\lambda(u^2 + d^2) - 2\beta s & -\beta\sqrt{2}(u+d) \\ -\beta\sqrt{2}(u+d) & \bar{\mu}^2 + 4\lambda s^2 \end{pmatrix}, \quad (5)$$

$$m_{a_0^+}^2 = \bar{\mu}^2 + 4\lambda(u^2 + d^2 + ud) - 2\beta s, \quad (6)$$

$$m_{\kappa^+}^2 = \bar{\mu}^2 + 4\lambda(u^2 + s^2 + su) - 2\beta d, \quad (7)$$

$$\begin{pmatrix} m_\sigma^2 \\ m_{f_0}^2 \end{pmatrix} = \text{eigv} \quad (8)$$

$$\times \begin{pmatrix} \bar{\mu}^2 + 4\lambda'(u+d)^2 + 6\lambda(u^2 + d^2) + 2 + \beta s & (4\lambda' s + \beta)\sqrt{2}(u+d) \\ (4\lambda' s + \beta)\sqrt{2}(u+d) & \bar{\mu}^2 + 8\lambda' s^2 + 12\lambda s^2 \end{pmatrix},$$

$$\phi^{s\bar{s}-\eta'} = \frac{1}{2} \arctan \frac{-2\sqrt{2}\beta(u+d)}{2\lambda(u^2 + d^2 - 2s^2) - 2\beta s}, \quad (9)$$

$$\begin{aligned} \phi^{s\bar{s}-f_0} &= \frac{1}{2} \arctan \\ &\times \frac{2\sqrt{2}(4\lambda' s + \beta)(u+d)}{4\lambda'[(u+d)^2 - 2s^2] + 6\lambda(u^2 + d^2 - 2s^2) + 2\beta s}, \quad (10) \end{aligned}$$

where “eigv” means the eigenvalues of the matrix which follows.

Let us first discuss the flavor symmetric limit $u = d = s$. Then the pseudoscalar decay constants are equal, $f_P = 2^{1/2}u = f_\pi = f_K$, while the mixing angles $\Theta^{\eta' \text{-singlet}} = \phi^{s\bar{s}-\eta'} - \arctan \sqrt{2}$ and $\Theta^{\sigma \text{-singlet}} = \phi^{s\bar{s}-\sigma} + \arctan(1/\sqrt{2})$ vanish, and one has 4 nondegenerate physical masses for the octet and singlet mesons $m_{P8}, m_{S8}, m_{P0}, m_{S0}$. Then there are also simple relations between the 5 model parameters $\lambda, \lambda', \bar{\mu}^2, \beta$ and $u = d = s$ and the 4 physical masses and the decay constant f_P :

$$\lambda = (3m_{S8}^2 - 2m_{P0}^2 - m_{P8}^2)/(12f_P^2), \quad (11)$$

$$\lambda' = (m_{S0}^2 - m_{S8}^2 - m_{P8}^2 + m_{P0}^2)/(12f_P^2), \quad (12)$$

$$\bar{\mu}^2 = (-3m_{S8}^2 + 5m_{P8}^2 + 4m_{P0}^2)/6, \quad (13)$$

$$\beta = (m_{P8}^2 - m_{P0}^2)/(3\sqrt{2}f_P), \quad (14)$$

$$u = d = s = \langle \sigma_{u\bar{u}} \rangle = \langle \sigma_{d\bar{d}} \rangle = \langle \sigma_{s\bar{s}} \rangle = f_P/\sqrt{2}. \quad (15)$$

It is obvious that we can reparametrize the theory in terms of these physical quantities, which can be kept fixed in the renormalization, as long as flavor symmetry is exact. (If one chooses the same tree-level values for the parameters λ, μ^2, β as found below in (16) and $\lambda' = 1$ but for $u = d = s$ the average value, 75.05 MeV , or $f_P = 106 \text{ MeV}$, one would have $m_{P8} = 384 \text{ MeV}$, $m_{P0} = 956 \text{ MeV}$, $m_{S8} = 1086 \text{ MeV}$, $m_{S0} = 741 \text{ MeV}$.) Thus these masses can be thought of as already “unitarized”. On the other hand the original parameters and the induced tri-meson couplings will be renormalized from the tree-level values.

Now breaking the flavor symmetry ($s \neq u = d$) we have only one more parameter, given by $f_K - f_\pi$, and it is evident that this breaking splits the degeneracy in the mass spectrum from 4 independent masses to 8 masses and generates two mixing angles. Thus one gets several tree-level predictions in particular for the scalars (Table 1). Of course now we expect these tree-level predictions to receive corrections from the unitarization, but for small symmetry breaking (experimentally $(s - u)/s \approx 0.308$) one would expect these corrections not to be larger than this, i.e. $< 30\%$.

Table 1. Predicted masses in MeV and mixing angles for two values of the λ' parameter. The asterisk means that m_π, m_K and $m_\eta^2 + m_{\eta'}^2$ are fixed by experiment together with $f_\pi = 92.42$ MeV and $f_K = 113$ MeV

Quantity	Model $\lambda' = 1$	Model $\lambda' = 3.75$	Experiment
m_π	137*	137*	137 [3]
m_K	495*	495*	495 [3]
m_η	538*	538*	547.3 [3]
$m_{\eta'}$	963*	963*	957.8 [3]
$\Theta^{\eta' - \text{singlet}}$	-5.0°	-5.0°	$(-16.5 \pm 6.5)^\circ$ [3]
m_{a_0}	1028	1028	983 [3]
m_κ	1123	1123	1430 [3]
m_σ	651	619	400–1200 [3]
m_{f_0}	1229	1188	980 [3]
$\Theta^{\sigma - \text{singlet}}$	21.9°	32.3°	$(28 - i8.5)^\circ$ [9]

We can fix 5 of the 6 parameters, leaving λ' free, by the 5 experimental quantities from the pseudoscalar sector alone: $m_\pi, m_K, m_\eta^2 + m_{\eta'}^2, f_\pi = 92.42$ MeV and $f_K = 113$ MeV [3], which all are accurately known from experiment. One finds that at the tree level

$$\begin{aligned} \lambda &= 11.57, \quad \bar{\mu}^2 = 0.1424 \text{ GeV}^2, \quad \beta = -1701 \text{ MeV}, \\ u = d &= 65.35 \text{ MeV}, \quad s = 94.45 \text{ MeV}. \end{aligned} \quad (16)$$

The remaining λ' parameter changes only the σ and f_0 masses and their trilinear couplings, not those of the pseudoscalars. Gavin et al. [6] ask for a group theoretical reason for this simplification. In fact, graphically it is easy to see that the λ' term can only break the OZI rule for the scalars, which can couple to the vacuum, but for the pseudoscalars the λ' term must leave the OZI rule intact. Therefore, λ' and μ affect the pseudoscalars only through the combination $\bar{\mu}^2 = \mu^2 + 4\lambda'(u^2 + d^2 + s^2)$.

It is of some interest that the simple Gell-Mann–Okubo mass formula for the mixing does not give the same result for the mixing angle between η and η' as our model. This is because there is flavor symmetry breaking also in the anomaly terms βd and βs in (3)–(10) above. E.g. for the octet pseudoscalar mass one gets from (3)–(10) $(4m_K^2 - m_\pi^2)/3 = 542.5$ MeV and a mixing angle of $\Theta^{s\bar{s}-\eta'} = -12.7^\circ$. This would be closer to the conventional mixing angle (-10° to -23°) [3], than our model -5.0° , where the octet η_8 mass is 566.1 MeV.

Some of the couplings of σ and f_0 depend sensitively on λ' , since λ' changes the small ideal mixing angle, $\phi^{s\bar{s}-f_0}$. It turns out below that λ' must be small, compared to λ , in order to fit the trilinear couplings. By putting $\lambda' = 1$ one gets a reasonable compromise for most of these couplings. With $\lambda' \approx 3.75$ one almost cancels the OZI rule breaking coming from the determinant term, and the scalar mixing becomes near ideal (for $\lambda' = -\beta/(4s) = 4.5$ the cancellation is exact).

As can be seen from Table 1 the predictions are not far from the experimental masses taken as $a_0(980)$, $f_0(980)$, $K_0^*(1430)$, and $\sigma(500)$. For a discussion of the existence

of the $\sigma(500)$ see my recent Frascati talk [4], which also includes some preliminary results of the present paper. Considering that one expects from our previous UQM analysis of the scalars [9] and our discussion above that unitarity corrections can have magnitudes of up to 30% and should go in the right direction compared to experiment, one must conclude that these results are even better than expected. Similar mass analyses as in Table 1 have been done in [6,7], although with somewhat different input data.

4 Trilinear couplings at the tree level

The trilinear coupling constants follow from the Lagrangian after one has made the shift $\Sigma \rightarrow \Sigma + V$. The most important spp couplings are at the tree level, when expressed in terms of the original parameters:

$$g_{\kappa+K^0\pi^+} = 4\lambda(d - u + s) - 2\beta, \quad (17)$$

$$\begin{aligned} g_{\kappa+K+\eta} &= -4\lambda u \sin \phi^{s\bar{s}-\eta'} + (2\sqrt{2}\lambda s + \beta\sqrt{2}) \\ &\quad \times \cos \phi^{s\bar{s}-\eta'}, \end{aligned} \quad (18)$$

$$\begin{aligned} g_{\kappa+K+\eta'} &= 4\lambda u \cos \phi^{s\bar{s}-\eta'} + (2\sqrt{2}\lambda s + \beta\sqrt{2}) \\ &\quad \times \sin \phi^{s\bar{s}-\eta'}, \end{aligned} \quad (19)$$

$$\begin{aligned} g_{\sigma\pi^+\pi^-} &= 2\sqrt{2} \cos \phi^{s\bar{s}-f_0} (u + d) [\lambda + 2\lambda'] \\ &\quad - \sin \phi^{s\bar{s}-f_0} (8\lambda' s + 2\beta), \end{aligned} \quad (20)$$

$$\begin{aligned} g_{\sigma K+K^-} &= \cos \phi^{s\bar{s}-f_0} \sqrt{2} [\lambda(4u - 2s) + 4\lambda'(u + d) + \beta] \\ &\quad + 4 \sin \phi^{s\bar{s}-f_0} [\lambda(u - 2s) - 2\lambda' s], \end{aligned} \quad (21)$$

$$\begin{aligned} g_{f_0\pi^+\pi^-} &= 2\sqrt{2} \sin \phi^{s\bar{s}-f_0} (u + d) [\lambda + 2\lambda'] \\ &\quad + \cos \phi^{s\bar{s}-f_0} (8\lambda' s + 2\beta), \end{aligned} \quad (22)$$

$$\begin{aligned} g_{f_0 K+K^-} &= \sin \phi^{s\bar{s}-f_0} \sqrt{2} [\lambda(4u - 2s) + 4\lambda'(u + d) + \beta] \\ &\quad - 4 \cos \phi^{s\bar{s}-f_0} [\lambda(u - 2s) - 2\lambda' s], \end{aligned} \quad (23)$$

$$g_{a_0\pi\eta} = \cos \phi^{s\bar{s}-\eta'} 2\sqrt{2}\lambda(u + d) - 2\beta \sin \phi^{s\bar{s}-\eta'}, \quad (24)$$

$$g_{a_0\pi\eta'} = \sin \phi^{s\bar{s}-\eta'} 2\sqrt{2}\lambda(u + d) + 2\beta \cos \phi^{s\bar{s}-\eta'}, \quad (25)$$

$$g_{a_0 K+K^-} = \sqrt{2} [\lambda(4u - 2s) - 4\lambda'(d - u) - \beta]. \quad (26)$$

In fact these can be written in more useful forms in terms of the predicted physical masses and mixing angles and decay constants:

$$g_{\kappa+K^0\pi^+} = (m_\kappa^2 - m_\pi^2)/(\sqrt{2}f_K), \quad (27)$$

$$\begin{aligned} g_{\kappa+K+\eta} &= -\sqrt{3} \sin(\phi^{s\bar{s}-\eta'} - 35.26^\circ) \\ &\quad \times (m_\kappa^2 - m_\eta^2)/(2f_K), \end{aligned} \quad (28)$$

$$\begin{aligned} g_{\kappa+K+\eta'} &= \sqrt{3} \cos(\phi^{s\bar{s}-\eta'} - 35.26^\circ) \\ &\quad \times (m_\kappa^2 - m_{\eta'}^2)/(2f_K), \end{aligned} \quad (29)$$

$$g_{\sigma\pi^+\pi^-} = \cos \phi^{s\bar{s}-f_0} (m_\sigma^2 - m_\pi^2)/f_\pi, \quad (30)$$

$$\begin{aligned} g_{\sigma K+K^-} &= -\sqrt{3} \sin(\phi^{s\bar{s}-f_0} - 35.26^\circ) \\ &\quad \times (m_\sigma^2 - m_K^2)/(2f_K), \end{aligned} \quad (31)$$

$$g_{f_0\pi^+\pi^-} = \sin \phi^{s\bar{s}-f_0} (m_{f_0}^2 - m_\pi^2)/f_\pi, \quad (32)$$

$$g_{f_0 K+K^-} = \sqrt{3} \cos(\phi^{s\bar{s}-f_0} - 35.26^\circ)$$

Table 2. Predicted couplings $\sum_i g_i^2/4\pi$ (in GeV^2), when $\lambda' = 1$, compared with experiment and predicted widths with experiment (in MeV). (We have used isospin invariance to get the sum over charge channels, when there are data for one channel only.) The predicted $f_0 \rightarrow \pi\pi$ width is extremely sensitive to the value of λ' (for $\lambda' = 3.75$ it nearly vanishes) and unitarity effects as discussed in the text. Also the $a_0\pi\eta$ coupling is very sensitive to loop corrections due to the $K\bar{K}$ threshold

Process	$\sum_i g_i^2/4\pi$ in model	$\sum_i g_i^2/4\pi$ in experiment	$\sum_i \Gamma_i$ model	$\sum_i \Gamma_i$ experiment
$\kappa^+ \rightarrow K^0\pi^+ + K^+\pi^0$	7.22	-	678	278 ± 23 [3, 10]
$\kappa^+ \rightarrow K^+\eta$	0.28	≈ 0 [10]	13	< 26 [10]
$\sigma \rightarrow \pi^+\pi^- + \pi^0\pi^0$	2.17	1.95 [11]	574	300–1000 [3]
$\sigma \rightarrow K^+K^- + K^0\bar{K}^0$	0.16	0.004 [11]	0	0
$f_0 \rightarrow \pi^+\pi^- + \pi^0\pi^0$	1.67	$0.765_{-0.14}^{+0.20}$ [12]	see text	40–100 [3]
$f_0 \rightarrow K^+K^- + K^0\bar{K}^0$	6.54	$4.26_{-1.12}^{+1.78}$ [12]	0	0
$a_0^+ \rightarrow \pi^+\eta$	2.29	0.57 [12]	273 see text	50–100 [3]
$a_0^+ \rightarrow K^+\bar{K}^0$	2.05	$1.34_{-0.28}^{+0.36}$ [12]	0	0

$$\times (m_{f_0}^2 - m_K^2)/(2f_K), \quad (33)$$

$$g_{a_0\pi\eta} = \cos \phi^{s\bar{s}-\eta'} (m_{a_0}^2 - m_\eta^2)/f_\pi, \quad (34)$$

$$g_{a_0\pi\eta'} = \sin \phi^{s\bar{s}-\eta'} (m_{a_0}^2 - m_{\eta'}^2)/f_\pi, \quad (35)$$

$$g_{a_0K^+K^-} = (m_{a_0}^2 - m_K^2)/f_K. \quad (36)$$

where we inserted $\arctan(1/\sqrt{2}) = 35.26^\circ$. In Table 2 8 different spp couplings are compared with quoted experimental numbers. In some of the channels of Table 2 the resonance is below threshold and the widths therefore vanish at the resonance mass. However, the coupling constants have recently been determined through a loop diagram from $\phi \rightarrow K\bar{K} \rightarrow \gamma\pi\pi$ and $\phi \rightarrow K\bar{K} \rightarrow \gamma\pi\eta$ (albeit in a somewhat model dependent way) by the Novosibirsk group [11, 12]. For channels where the phase space is large, it is important that one includes a form factor related to the finite size of physical mesons. In the 3P_0 quark pair creation model a radius of ≈ 0.8 fm leads to a gaussian form factor, as in the formula below, where $k_0 \approx 0.56$ GeV/c (as was found in the UQM [9]). Thus the widths are computed from the formula

$$\Gamma(m) = \sum_{\text{isospin}} \frac{g_i^2}{8\pi} \frac{k^{cm}(m)}{m^2} e^{-[k^{cm}(m)/k_0]^2}. \quad (37)$$

As can be seen from Table 2 most of the couplings have magnitudes not far from the experimental ones. Only the $f_0 \rightarrow \pi\pi$ and $a_0 \rightarrow \pi\eta$ couplings and widths come out a bit large, but these are very sensitive to higher order loop corrections due to the $K\bar{K}$ threshold, and $f_0 \rightarrow \pi\pi$ is extremely sensitive to the scalar near-ideal mixing angle and λ' . If one chooses $\lambda' = 3.75$ this mixing angle nearly vanishes ($\phi^{s\bar{s}-f_0} = -3.0^\circ$) together with the $f_0 \rightarrow \pi\pi$ coupling (c.f. (32)). From our experience with the UQM [9] the $a_0 \rightarrow K\bar{K}$ peak width, when unitarized, is reduced, because of the $K\bar{K}$ threshold, by a factor of up to 5. Therefore one cannot expect that the tree-level couplings should agree better with the data than what those of Table 2 do. After all, this is a very strong coupling model ($\lambda = 11.57$,

leading to large $g_i^2/4\pi$) and higher order effects should be important.

5 Conclusions

In summary, I find that the linear sigma model with three flavors, at the tree level, works much better than what is generally believed. When the 6 model parameters are fixed mainly by the pseudoscalar masses and decay constants, one predicts the 4 scalar masses and mixing angle to be near those of the experimentally observed nonet $a_0(980)$, $f_0(980)$, $\sigma(500)$, $K_0^*(1430)$. Also 8 couplings/widths of the scalars to two pseudoscalars are predicted reasonably close to their presently known, rather uncertain experimental values. The agreement is good enough considering that some of these are expected to have large higher order corrections. The model works, in my opinion, just as well as the naive quark model works for the heavier nonets. A more detailed data comparison would become meaningful, after one has included higher order effects, i.e. after one has unitarized the model, e.g., along the lines of the UQM [9].

Those working on chiral perturbation theory and non-linear sigma models usually point out that the linear model does not predict all low energy constants correctly. However, one should remember that the energy regions of validity are different for the two approaches. Chiral perturbation theory usually breaks down when one approaches the first scalar resonance. The linear sigma model, on the other hand, includes the scalars from the start and can be a reasonable interpolating model in the intermediate energy region near 1 GeV, where QCD is too difficult to solve.

These results strongly favor the interpretation that the $a_0(980)$, $f_0(980)$, $\sigma(500)$, and $K_0^*(1430)$ belong to the same nonet, and that they are the chiral partners of the π , η , η' , and K . If the latter are believed to be unitarized $q\bar{q}$ states, so are the light scalars $a_0(980)$, $f_0(980)$, $\sigma(500)$,

and $K_0^*(1430)$, and the broad $\sigma(500)$ should be interpreted as an existing resonance. The σ is a very important hadron indeed, as is evident in the sigma model, because this is the boson which gives the constituent quarks most of their mass and thereby it gives also the light hadrons most of their mass. Therefore, it is natural to consider the $\sigma(500)$ as the Higgs boson of the strong interactions.

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